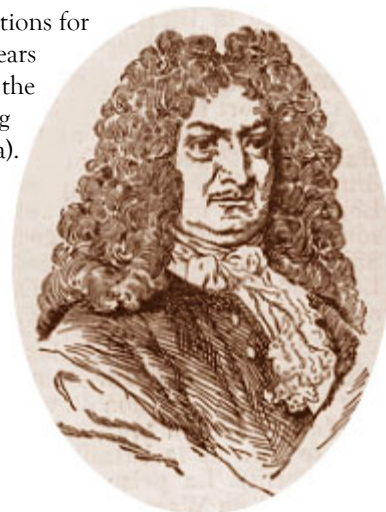


ORIGINS OF CALCULUS NOTATIONS

Gottfried Wilhelm Leibniz (1646-1716) first introduced the modern notations for integration and differentiation in his private notes of 1675. In the two years previous, Leibniz had worked intensely on many complex problems, including the rectification of curves (arc length) and inverse tangents (antiderivatives). During his work, Leibniz realized both problems could be reduced to quadratures (area).

In 1673, Leibniz wrote:

“The two questions, the first that of finding the description of the curve from its elements, the second that of finding the figure from the given differences, both reduce to the same thing. From this fact it can be taken that almost the whole of the theory of the inverse method of tangents is reducible to quadratures.”



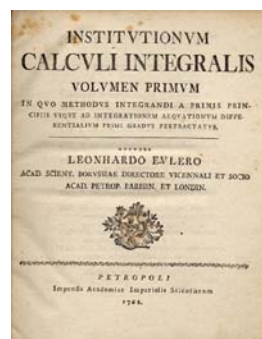
In working with the tangent problem, Leibniz used the notation *omn*, an abbreviation for *omnis* [from Latin: all] to indicate the sum of an area described by a function (interestingly, the term *function* was also of Leibniz, which he took in 1692 from the Latin *functio* meaning achievement or execution). A simplification effort intended to reduce mistakes eventually replaced the *omn* notation with a large ‘S’, which meant simply,

“sum”: \int

At the time, all integrals were assumed to be definite. Despite this, limits of integration were not indicated along side the integral symbol. Instead, they were notated only in words.



Leonhard Euler (1707-1783) was the first to publish a symbolic notation for the limits of integration in his *Institutionum calculi integralis*. In this work, Euler denoted integration limits in brackets and used the Latin words *ab* (from) and *ad* (to).



Jean Baptiste Joseph Fourier (1768-1830) finalized the modern notation for the definite integral in 1822. In his work, “*The Analytical Theory of Heat*”, Fourier wrote:

“Nous désignons en général par le signe \int_b^a l'intégrale qui commence lorsque la variable équivaut à a, et qui est complète lorsque la variable équivaut à b”

“We in general indicate by the sign \int_b^a the integral which starts when the variable is equivalent to a, and which is complete when the variable is equivalent to b”

– Jean Baptiste Joseph Fourier (1768-1830), “*The Analytical Theory of Heat*” 1822.



REFERENCES

Cabillon, Professor Julio Gonzalez; Montevideo, Uruguay: “Historia Matematica.” Internet: “<http://chasque.net/jgc/>”. 2003.

Cajori, Florian, Professor of Mathematics, Colorado College, Colorado Springs, CO, 1859-1930: “A History of Mathematical Notations.” Dover, (1928, 29) 1993, 451+367 pp.

Deborah Hughes-Hallett, University of Arizona; Andrew M. Gleason, Harvard University; William G. McCallum, University of Arizona; et al.: “Calculus: Single and Multivariable, Third Edition” John Wiley & Sons, Inc. ISBN 0471-40827-1. 2002.

Mehl, Serge: “Chronomath; Notations & Symboles.”. Internet: “<http://www.sciences-en-ligne.com/momo/chronomath/base/notations.html>”. 2001.

Miller, Jeff: “History of Mathematics Pages; Earliest Uses of Various Mathematical Symbols; Symbols used in calculus.” Internet: “<http://members.aol.com/jeff570/>”. 2001.